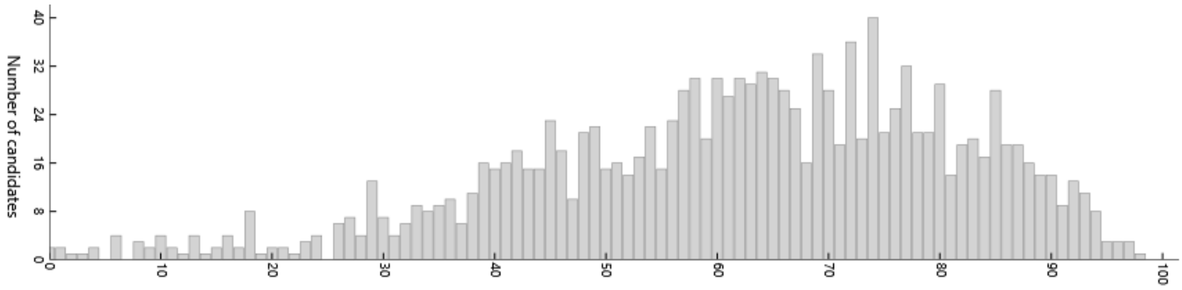
Government of **Western Australia**
School Curriculum and Standards Authority

2022 ATAR course examination report: Mathematics Specialist

Year	Number who sat	Number of absentees
2022	1350	28
2021	1503	18
2020	1526	23
2019	1435	32

The number of candidates sitting and the number attempting each section of the examination can differ as a result of non-attempts across sections of the examination.

Examination score distribution–Written



Summary

Attempted by 1348 candidates Mean 61.78% Max 97.76% Min 0.00%

Section means were:

Section One: Calculator-free	Mean 71.14%		
Attempted by 1348 candidates	Mean 24.90(/35)	Max 35.00	Min 0.00
Section Two: Calculator-assumed	Mean 56.75%		
Attempted by 1346 candidates	Mean 36.88(/65)	Max 63.49	Min 0.00

General comments

The 2022 Mathematics Specialist examination provided candidates with many opportunities to demonstrate their knowledge of standard techniques and related concepts. The mean score of 61.78% compared favourably with the 2021 mean of 60.77%. The Calculator-free section was well received, as it provided some straightforward questions testing standard techniques. The Calculator-assumed section provided an appropriate balance of questions testing routine concepts and questions requiring a deeper level of understanding. The mean of 56.75% for the Calculator-assumed section perhaps reflected candidates not having the depth of understanding required.

Questions such as Question 6 part (c) and Question 19 part (c) required candidates to show a greater depth of understanding of some concepts. The distribution of marks indicated a very diverse cohort. There were a number of candidates that were not able to respond to many questions.

The length of the paper appeared to be appropriate. There was a significant decrease in the number of candidates who attempted to answer Question 19 part (c), though this may have been due to an inability to answer the question, rather than a time factor.

Advice for candidates

- Show a clear sequence of ideas and write a clear conclusion. Markers should not be expected to search for the 'answer', or to construct meaning out of a solution for themselves.
- Check your work from one line to another. An example of this is omitting a negative sign in the next line of working.
- Be specific in explanations in the context of the question and refer directly to the relevant key words. Do not use the words 'it' or 'they', as this is meaningless.
- Ensure you include absolute value brackets when writing a natural logarithm anti-derivative.

Advice for teachers

- Give students practise in solving algebraic inequalities.
- Develop students' understanding in 3D vectors to a more conceptual level. In particular, the case where three planes do not intersect in a unique point is an area of concern.

Comments on specific sections and questions

Section One: Calculator-free (48 Marks)

Candidates were able to answer the standard technique style of questions well. This was observed in:

- sketching the graph of the reciprocal of a function (Question 2 part (b))
- evaluating a definite integral using trigonometric identities (Question 3)
- integration using partial fractions (Question 4 part (b)).

Areas that were seen to cause difficulties were:

- determining the domain for the composition of a function (Question 1 part (b))
- writing the solution for the case where planes intersect in a line (Question 5 part (b)).

In Question 5 part (b), most candidates did not correctly specify the solutions in the case of planes intersecting in a line. This was true even for many of the higher scoring candidates. It was very common for candidates to cease working once they had stated that there were infinitely many solutions.

Question 1 attempted by 1345 candidates Mean 4.12(/6) Max 6 Min 0

Part (a) was generally answered well, although a number of candidates believed that $\sqrt{9} = \pm 3$. There is only one value for $\sqrt{9}$ and that is a positive number. In finding the domain for $f\left(g\left(x\right)\right)$, many candidates had significant difficulty in solving the inequality $4 - \frac{1}{x^2} \geq 0$ and in many cases they attempted to avoid this issue by solving $4 - \frac{1}{x^2} = 0.$ This then created the issue of knowing what to write after stating $x = \pm\frac{1}{2}$. Most candidates knew the meaning of what constituted a one-to-one function. Some candidates confused the question as to whether g^{-1} was a function.

Question 2 attempted by 1340 candidates Mean 5.17(/7) Max 7 Min 0

In part (a), those candidates who drew the graph of $y = \left|f\left(x\right)\right|$, and considered the intersection with $y = x$, were generally successful. Overall, there was excellent performance in candidates' drawing the reciprocal of a function and most appeared to be well-prepared for this question.

Question 3 attempted by 1344 candidates Mean 4.62(/5) Max 5 Min 0
Candidates generally handled the expansion of $(\sin x + \cos x)^2$ well and were able to use the suggested trigonometric identities to integrate correctly.

Question 4 attempted by 1339 candidates Mean 7.07(/8) Max 8 Min 0
The use of partial fractions was generally well done. The use of the natural logarithm of an absolute value function was also done well, with candidates being rewarded for process. Many candidates often went on to incorrectly use logarithm properties to express the answer as a single logarithm, though this was not penalised as it was not a behaviour in the marking key.

Question 5 attempted by 1340 candidates Mean 3.20(/6) Max 6 Min 0
There was a large range in vocabularly witnessed for part (a). Many candidates wrote down vectors, not stating that they were the normal vectors, while other responses were vague and confusing. Precise language was required for a rather simple idea. In part (b), the majority of candidates could not respond appropriately for the case where three planes intersect in a line. The common response was to leave the answer at 'there are infinitely many solutions', as though the question asked how many solutions there were. The question posed was to solve the equations simultaneously, so in essence it is necessary to express the answer as the equation for a line, using some parameter. In part (c), candidates were able to describe the intersection as a line in space, for those that obtained infinitely many solutions in part (b). This should have reinforced that, in part (b), the equation for the line needed to be specified using some parameter, or at least candidates should have written relationships between the variables x, y, z.

Question 6 attempted by 1338 candidates Mean 5.04(/8) Max 8 Min 0
In part (a), interpreting the correct polar form was handled well by most candidates. The limiting behaviour for many was the inability to read the polar angular scale. The use of the complex number properties in polar form was well done in part (b). The first issue for some candidates was to correctly add fractions to obtain $\frac{19\pi}{12}$, and secondly to re-write this as $-\frac{5\pi}{12}$. Those that did obtain $-\frac{5\pi}{12}$ then could not always position this correctly (confusing it with $-\frac{7\pi}{12}$), or they simply omitted to plot the position for w. Many candidates found part (c) challenging and the question required a logical sequence of thought. Candidates needed to deduce a value for n (or k) rather than simply stating $n = 12$.

Question 7 attempted by 1324 candidates Mean 4.94(/8) Max 8 Min 0
The implicit differentiation in part (a) was generally handled well, with a small proportion indicating some confusion with the variable by writing $2 = \sec^2(x).y'$. Perhaps the use of the notation y' caused candidates to think that the independent variable was x. Some candidates omitted finding the equation of the tangent. Those that opted to express the area in terms of y found this straightforward. There were many correct alternatives for writing the area in terms of x but these were often not accompanied by the correct limits or the correct

integrand. Part (c) proved difficult for candidates that wrote the area in terms of x, who then realised that they did not know the anti-derivative for $\tan^{-1}(2x)$. Some candidates boldly wrote $\tan^{-1}(2x)=\frac{\cos 2x}{\sin 2x}$ which enabled them to be in a position to anti-differentiate $\frac{\cos 2x}{\sin 2x}$ as $\frac{1}{2}\ln|\sin 2x|$. Whilst these candidates could not achieve the first marking key behaviour, if the anti-differentiation was done correctly using the absolute value then the second marking key behaviour was awarded.

Section Two: Calculator-assumed (86 Marks)
The performance on the Calculator-assumed section did not match that of the Calculator-free work. The wider array of questions and concepts appeared to challenge a large number of candidates.

Difficulties were observed with:

- forming the correct expression for the area of a regular hexagon (Question 8)
- algebraically working with a complex number equation (Question 9 part (b))
- expressing a given locus equation correctly (Question 9 part (c))
- dealing with motion where the velocity is given as function of displacement (Question 11)
- using correct mathematics or vector notation (Questions 10 and 19).

Question 8 attempted by 1305 candidates Mean 2.48(/4) Max 4 Min 0
Some candidates had difficulty forming the correct area expression in terms of the side length. This aside, the question was well done. Candidates needed to include the correct units for the rate of change of the area in order to achieve full marks.

Question 9 attempted by 1325 candidates Mean 4.15(/8) Max 8 Min 0
In part (a), many candidates confused this locus question with the perpendicular bisector locus type question. Using a ruler to draw a line segment was an advantage. Many candidates performed poorly in part (b). A common error was to mistake that $(z+i)\left(\overline{z+i}\right)=2$ meant $(z+i)(z-i)=2$ and hence $z^2+1=2$ with $z^2=1$. These candidates then compounded their error by interpreting $z^2=1$ as an equation of a circle, resulting in zero marks for this question. In part (c), candidates appeared to be comfortable with this question, given its similarity to questions from past papers. The main error was to write the argument inequality from the correct point. This could have been either the origin or the point $(1,1)$. There were a few candidates who expressed the argument condition as $Arg\left(z-(1-i)\right)-Arg\left(z-(-1+i)\right)=\frac{\pi}{4}$ which comes from the use of the Central Angle Theorem.

Question 10 attempted by 1332 candidates Mean 5.89(/9) Max 9 Min 0
In part (a)(i), most candidates knew that this integral expression gave the change in displacement. A common error was to state ‘displacement after the first second’. In part (a)(ii), almost all candidates realised that this involved some distance, although very few realised that the time period happened to be the time taken to do one circuit of the path. For part (b) correct mathematics and vector notation was required and many candidates did not

demonstrate this aspect. It needed to be realised that the constant of integration in this case was a vector.

Question 11 attempted by 1335 candidates Mean 4.80(/8) Max 8 Min 0

Candidates who expressed velocity as the derivative $\frac{dx}{dt}$ where usually successful in part (b). Many candidates confused the variables that they were integrating with respect to. Those that completed part (b) well usually achieved well in part (c) also.

Question 12 attempted by 1326 candidates Mean 5.32(/9) Max 9 Min 0

Almost all candidates stated that the distribution for the sample mean was normal, but this was often marred by many stating that the mean of this distribution was $\overline{X}$ rather than μ. It was then unclear what value was being referred to for the standard deviation. For part (b), a minority of candidates realised that the event involved the combined area in the extremes of the normal distribution. Part (c) was a reasonably straighforward question yet some candidates wrote down irrelevant formula. Many wrote their answer as a confidence interval, which was not the stated question. In part (d), many candidates realised that the randomness of the sample was lost. Most found it more difficult to then state that all the assumptions for the normal distribution of the sample mean were invalidated.

Question 13 attempted by 1317 candidates Mean 5.07(/8) Max 8 Min 0

A large number of candidates showed that they could not distinguish between the given cubic equation and the equation of the curve that intersected the line $y=\sqrt{24}$. Many made comments about the x intercepts of the graph which was not relevant to the question. A small minority were able to make the correct observation as to why the complex solutions formed a conjugate pair. Part (b) was very straightforward given that a calculator could have been used to obtain the solutions. Despite trouble with part (a), candidates performed well in writing an expression for the area shaded. Errors were often made in what function was entered into the CAS calculator, or what limits of integration were entered.

Question 14 attempted by 1330 candidates Mean 3.97(/6) Max 6 Min 0

In part (a), many candidates did not know which formula to use while others used four as the half-width. Candidates needed to write the values that they used in calculations. If the critical z-value written was 2.58, this gave a different answer to $z=2.576$. Typically, candidates wrote $z=2.58$ but then the CAS value for z was used, so the answers did not match. Part (b) was straightforward, but many repeated themselves with comments such as 'the width of each interval could be different, along with different standard deviations'. Some candidates incorrectly stated that the sample sizes were different. In making a reasoned comment in part (c), a majority of candidates stated that the conclusion was not correct, but many could not specifically state why this was not correct. The fact that the interval was wider was because there was more variation in the sample mean for a size of 50. It was quite common for candidates to say that the confidence interval was 99% certain to contain μ. This notion is not correct. If samples of size 50 were repeatedly taken, it could then be expected that 99% of the confidence intervals contained the unknown value μ. It is not known that a single confidence interval from a sample of size 50 would contain μ.

Question 15 attempted by 1244 candidates Mean 1.74(/4) Max 4 Min 0

This question provided a different way to assess rational functions and their graphs. In this instance, the rational function had no horizontal intercepts or vertical asymptotes. Despite this, performance was quite good for some candidates. Those that used the completion of square form for a quadratic function had greater success.

Question 16 attempted by 1290 candidates Mean 2.16(/4) Max 4 Min 0
The instruction 'using the growth rate equation' was key in answering part (a). For part (b), performance was good in realising that at $t = 4$ the population was now well under half the new limiting value. A point of inflection needed to be evident and then the levelling off around $P = 300$ for full marks. Those that exhibited only a concave down curve were not able to achieve full marks, given that it did show a horizontal asymptote at $P = 300$.

Question 17 attempted by 1325 candidates Mean 6.64(/10) Max 10 Min 0
Candiates did well in calculating the volume of revolution, indicating that this concept is well understood. Common errors were not including the factor π, forgetting to enter the factor π into their CAS calculator or failure to quote appropriate units with their answer. In part (b), the technique of increments was done well. Many candidates thought they needed to find the volume from $y = 0$ to $y = 32$ to substitute for V to do part (c). The definition for $V(t)$ was clearly made such that for $y = 6$, $V(0) = 0$. The correct units for the rate of change answer was required in order to achieve full marks. In part (d), performance was rather mixed. Many candidates were not able to correctly determine the constant of integration, after performing the separation of variables.

Question 18 attempted by 1232 candidates Mean 3.46(/7) Max 7 Min 0
Most candidates made a fair effort in part (a), although many reverse engineered what needed to be shown. Most often it was not obvious how the candidate showed that $cis(-\theta) = \cos\theta - i\sin(\theta)$ which was important to show how the middle term became $-(2i\sin\theta)z^n$. In using the result from part (a), candidates needed to confront the issue of solving two separate complex equations. The second equation involving $z^3 = -cis\left(-\frac{\pi}{6}\right)$ caused considerable trouble with most simply ignoring the opposite sign and then attempting to solve the equation as if it was $z^3 = cis\left(-\frac{\pi}{6}\right)$.

Question 19 attempted by 1256 candidates Mean 3.13(/9) Max 9 Min 0
A high number of candidates attempted this question and did quite well. The main errors were forming a vector equation rather than the requested cartesian equation, or making an error in finding the cross product using their CAS calculator, most likely from entering the wrong digits. Those candidates that did attempt part (b) knew what they needed to do, in finding the line perpendicular to the plane. Various errors with accuracy crept in. Quite a few candidates found the shortest distance to the plane but then found it difficult to correctly use this answer to find the coordinates on the plane. Correct vector notation was not assessed in this question. Despite a very low success rate in part (c), there were many excellent and varied approaches taken to this question.